3-D Transformations

CS418 Computer Graphics
John C. Hart

3-D Affine Transformations

- In general

$$\begin{bmatrix} d & e & f & a \\ g & h & i & b \\ j & k & l & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dx + ey + fz + a \\ gx + hy + iz + b \\ jx + ky + lz + c \\ 1 \end{bmatrix}$$

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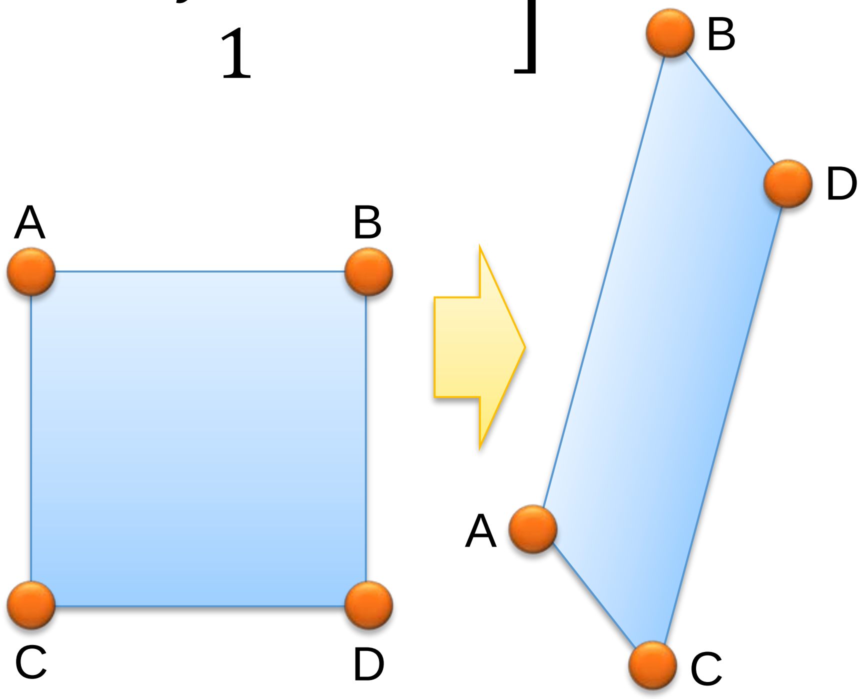
- Properties
  - Lines → Lines
  - Planes → Planes
  - Triangles → Triangles
  - Parallel → Parallel



3-D Affine Transformations

- Translation

$$\begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + a \\ y + b \\ z + c \\ 1 \end{bmatrix}$$

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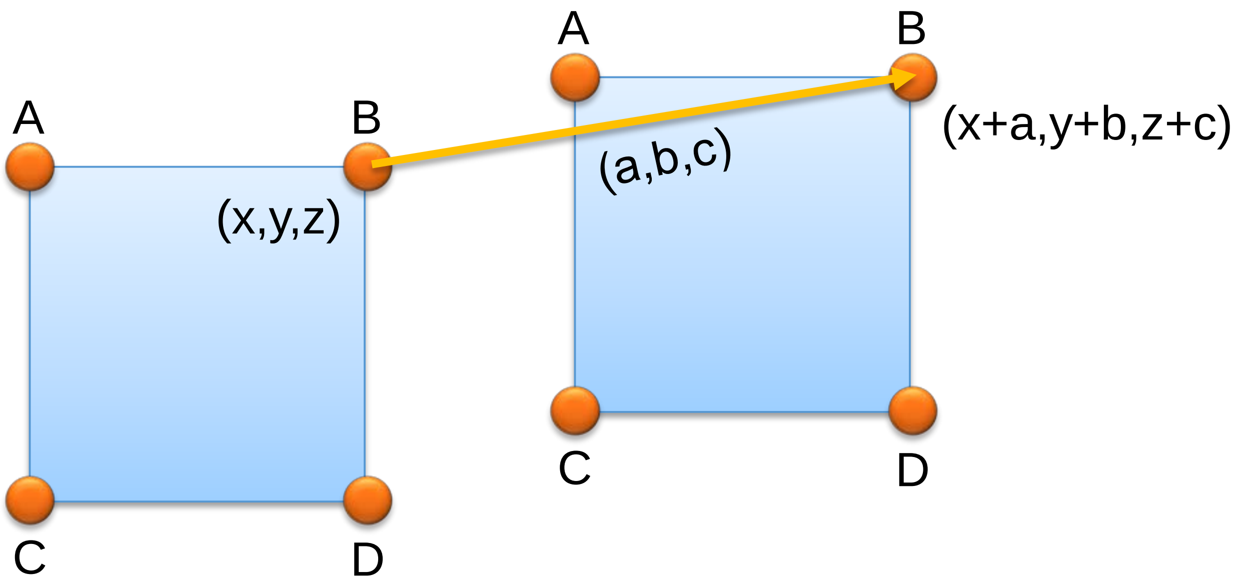

Scale

$$\begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} ax \\ by \\ cz \\ 1 \end{bmatrix}$$

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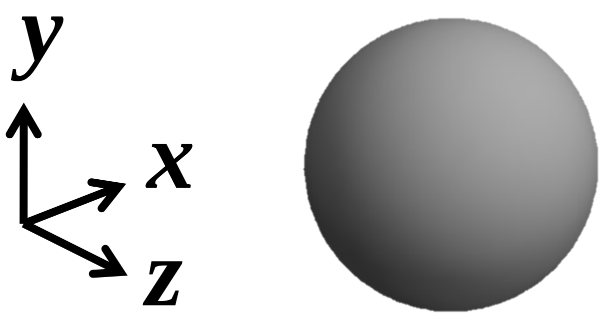

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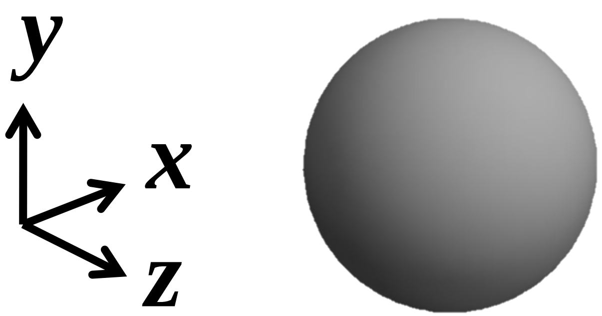
Uniform Scale
a = *b* = *c* = ¼



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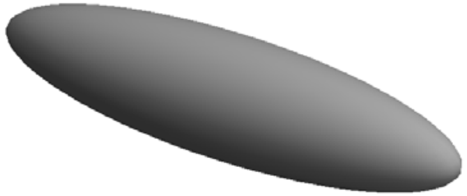
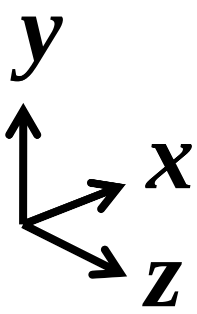


Stretch
$a = b = 1,\ c = 4$



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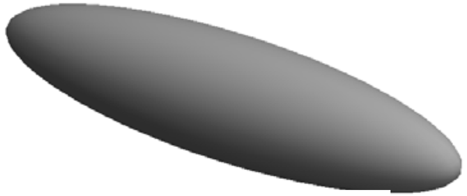
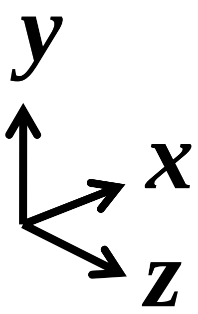


Stretch
a = *b* = 1, *c* = 4



Squash
a = *b* = 1, *c* = ¼

Scale

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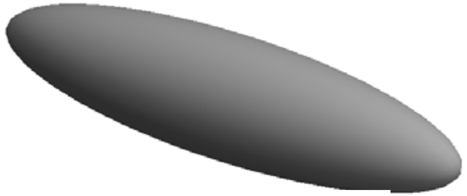

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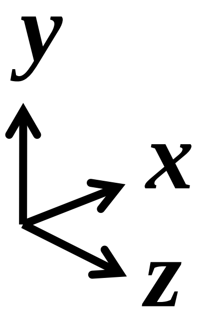

Squash
a = *b* = 1, *c* = ¼

Project
a = *b* = 1, *c* = 0

Inverse Transformations

- $M M^{-1} = M^{-1} M = I$

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- Inverse of a translation

$$\begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & & & -a \\ & 1 & & -b \\ & & 1 & -c \\ & & & 1 \end{bmatrix}$$

Inverse Transformations

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- Inverse of a translation

$$\begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & & & -a \\ & 1 & & -b \\ & & 1 & -c \\ & & & 1 \end{bmatrix}$$

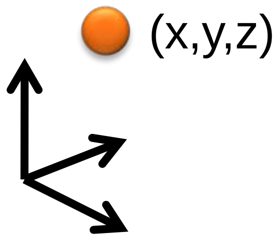
- Inverse of a scale

$$\begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix}^{-1} = \begin{bmatrix} {}^{1}\!/_{a} & & & \\ & {}^{1}\!/_{b} & & \\ & & {}^{1}\!/_{c} & \\ & & & 1 \end{bmatrix}$$

Points v. Vectors

- Points represented by 4-vector with homogeneous coordinate = 1

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



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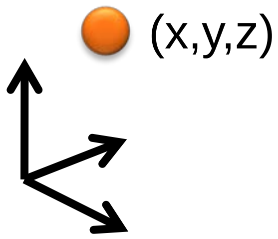
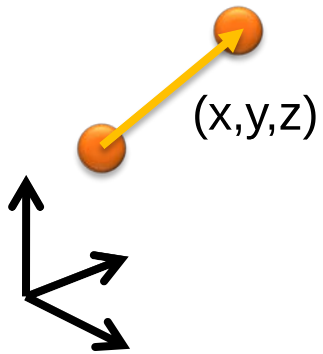


- Vectors represented by 4-vector with homogeneous coordinate = 0 $\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$



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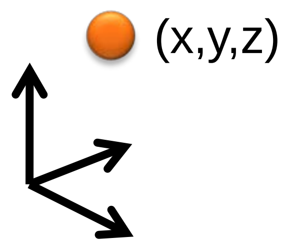
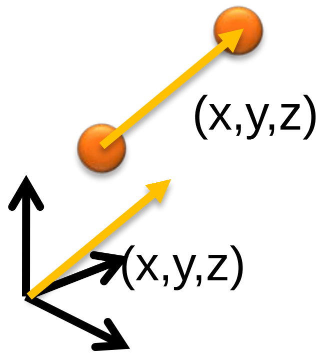


- Vectors represented by 4-vector with homogeneous coordinate = 0

$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$



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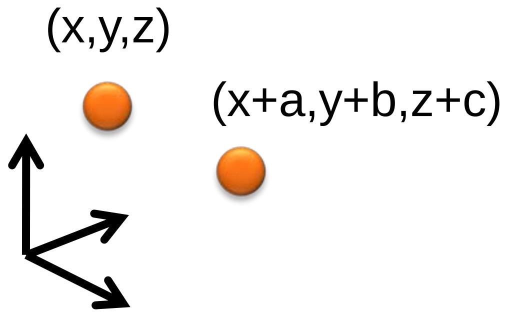

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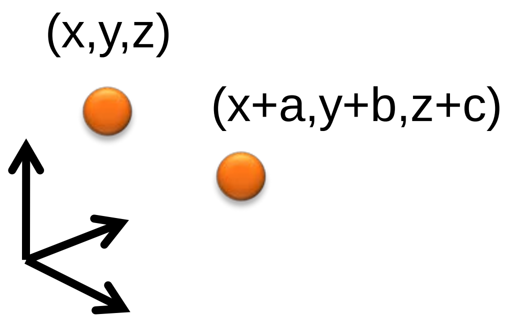

- Vectors don't

$$\begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$